Non-Gaussianity (NG) in CMB sky maps

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***(2017, Focus issue: Planck, CQG 34 Buchert France Steiner,* arXiv:1701.03347)**

- Introduction: the Cosmic Microwave Background as a weakly perturbed random field
- The interpretation of the temperature dipole
- Planck map Gaussianity versus sample Gaussianity
- Measuring and expanding the CMB NG a model-independent way
- Non-linear correction terms to NG

Conclusion

Introduction: the CMB as a random field

Very stringent prescriptions of the Standard Cosmological Model

CMB homogeneity, isotropy

Verified once original microwave observations are reduced

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CMB appears as a weakly perturbed temperature random field

when the dipole is suppressed:

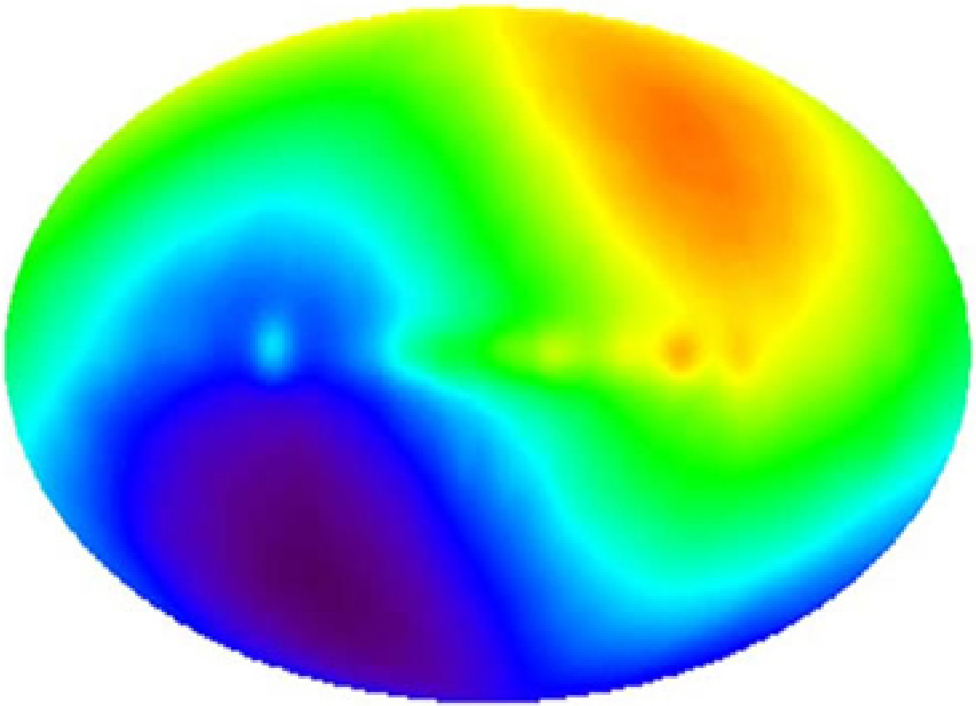
The interpretation of the dipole

$T_{dipole} = \pm(3{,}364.5 \pm 2.0 \mu K)$, (*2016 A&A 594 A8, Planck 2015 results - VIII*)*

once suppressed the dipole as 100% kinematic, the monopole is

$T_0 = (2{,}725{,}500.0 \pm 600.0 \mu K)$, *(*Planck 2015*)

but the CMB dipole can be not purely kinematic...



Planck Map Gaussianity versus ... Sample Gaussianity

The unique CMB temperature map of Planck is nearly Gaussian random field but ...

Central limit theorem (CLT) not fully testable,

CMB patched by ≈ 12732 independent regions (acoustic horizon) in the Standard Model

CMB map sample limited only by the number of maps

Our recent focus section article (arXiv:1701.03347) tests the CLT up to 100k ΛCDM model maps *(Planck 2015 cosmological parameters)*

Planck Map Gaussianity versus ... Sample Gaussianity

moments, **variances** and **morpho-statistical descriptors** characterize a CMB map,

averages of these quantities characterize the sample

we found, in agreement with the literature:

$\mu_{(Planck)} \approx -0.720\mu K$ VERSUS $\mu_{(sample)} \approx -0.001\mu K$,

min=-13.503μK, max=+13.958μK, standard deviation of $\mu_{(sample)}=3.208\mu K$.

$\sigma_0^{(Planck)} \approx 51.6\mu K$ VERSUS $\sigma_0^{(sample)} \approx 59.1\mu K$,

min=49.779μK, max=76.229μK, standard deviation of $\sigma_0^{(sample)}=3.108\mu K$.

Planck Map Gaussianity versus ... Sample Gaussianity

For the **morpho-statistical descriptors** that we use, the analytic predictions over a Gaussian random field are known, given by Tomita (*1986 Prog. Theor. Phys. n°75 and 76*) for the Minkowski functionals.

The discrepancy function ΔP of the PDF P is,

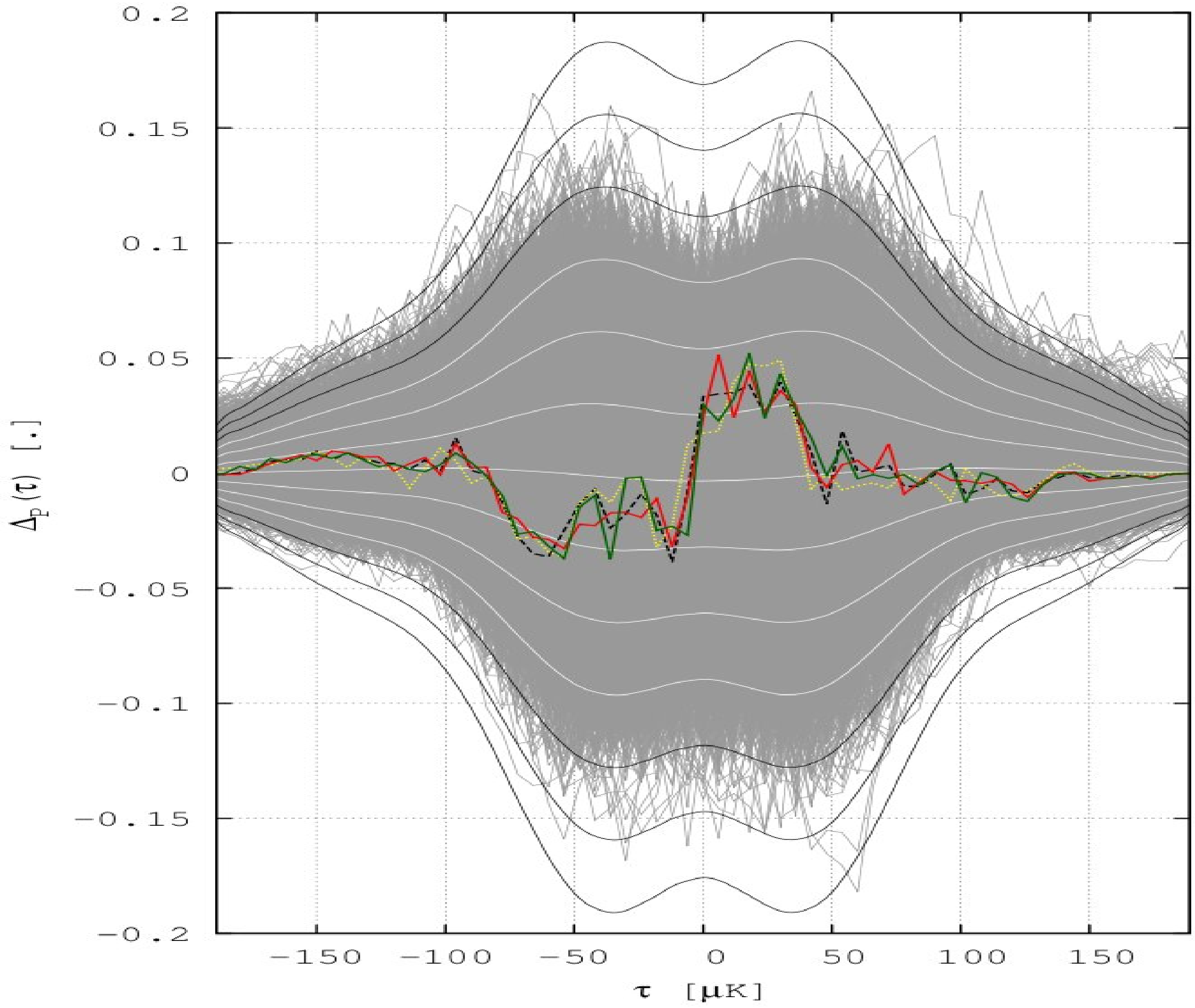
$$\Delta P(\tau) := \frac{P(\tau) - P^G(\tau)}{max(P^G)} \text{ with } P^G(\tau) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp \frac{-(\tau-\mu)^2}{2\sigma_0^2} .$$

The discrepancy function Δ_1 of the 2nd Minkowski functional v_1 is,

$$\Delta_1(\nu) := \frac{v_1(\nu) - v_1^G(\nu)}{max(v_1^G)} \text{ with } v_1^G(\nu) = \frac{1}{8\sqrt{2}} \frac{\sigma_1}{\sigma_0} e^{-\nu^2/2} .$$

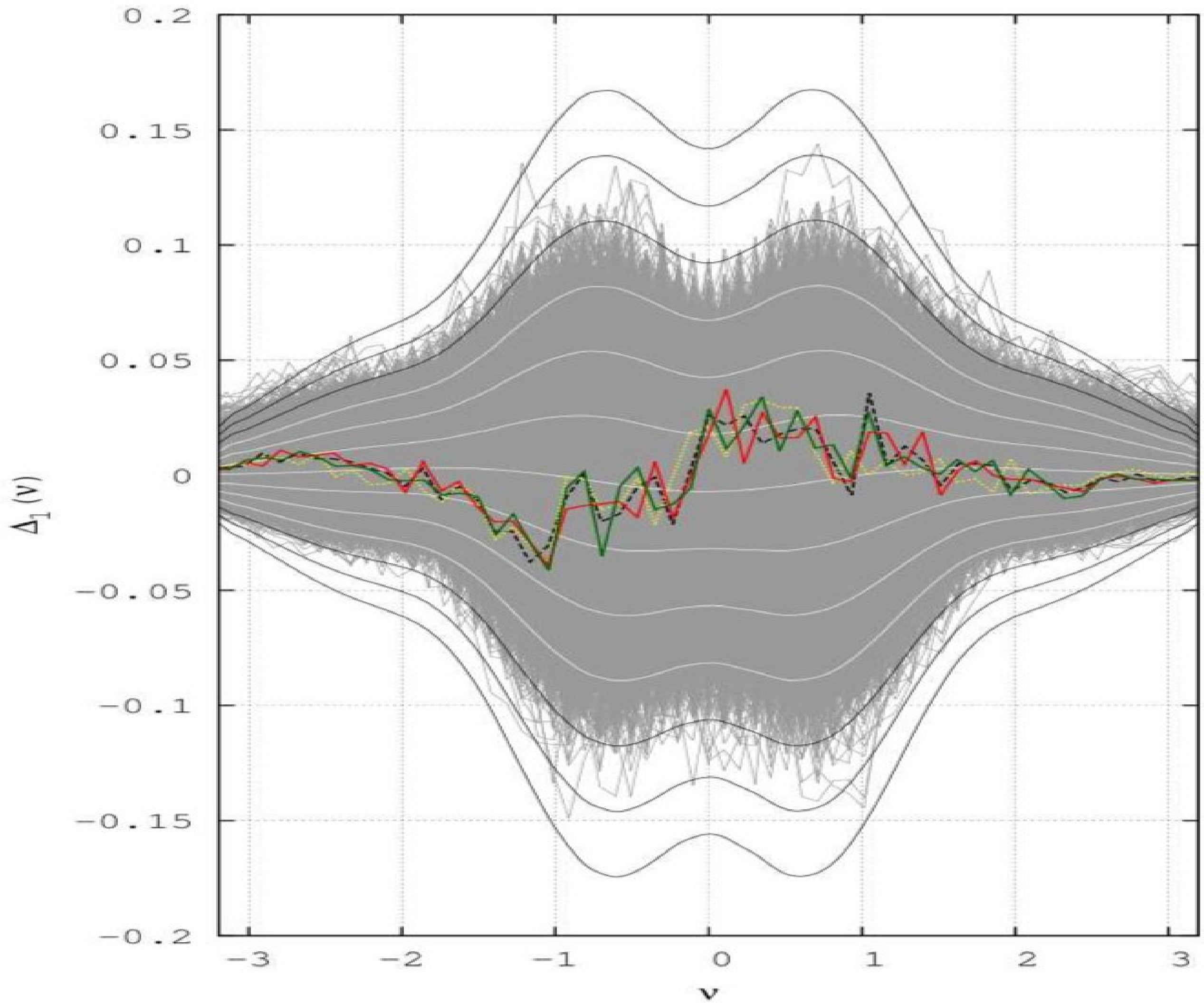
Planck Map Gaussianity versus ...
Sample Gaussianity

Discrepancy function of the PDF



Planck Map Gaussianity versus ... Sample Gaussianity

Discrepancy function of v_1, the 2nd Minkowski functional



Measuring and expanding CMB NG a model-independent way

does perturbation theory (PT) apply to the perturbed random field of the CMB?

We compare our model-independent methods to the perturbative ones (pioneering work on **Minkowski functionals**, *T Matsubara*) for the 100000 map sample with U73 mask,

- Expanding discrepancy function Δ_P in non-truncated Hermite expansions,

$$a_P(n) = \frac{1}{\sqrt{2\pi}} \int_{\nu_{min}}^{\nu_{max}} \Delta_P(\nu) He_n(\nu) d\nu, \;\; \Delta_P^{He}(\nu) = \mathrm{e}^{-\nu^2/2} \sum_{\mathrm{n}=3}^{\mathrm{n}=6} \frac{\mathrm{a_k(n)}}{\mathrm{n!}} \mathrm{He_n}(\nu).$$

- Versus Hierarchical ordering (HO) of the cumulants, $C_n \approx \sigma_0^{n-2} \; (n \geq 2)$, predicted in single field Inflation models. “This suggest to expand the discrepancy function into a power series (PT) in terms of σ_0”,

this gives finally:

$$\Delta_P^{HO(4)}(\nu) = \mathrm{e}^{-\nu^2/2} \sum_{\mathrm{n}=3}^{\mathrm{n}=12} \frac{\mathrm{a_P^{HO}(4, n)}}{\mathrm{n!}} \mathrm{He_n}(\nu).$$

Measuring and expanding CMB NG a model-independent way

does perturbation theory apply to the perturbed random field of the CMB?

In Hierarchical ordering (HO) of the cumulants the NG should be expanded in the vicinity of zero but it is done for $\sigma_0 \approx 59\mu K$, which is far from zero... This is why we test its validity.

RESULT: expanded at 2^{nd} order, the HO is not a perfect fit of the CMB non-Gaussianity. The HO should be at least 4^{th} order to fit the NG with the same precision as the model-independent method limited to the 2^{nd} order.

Non-linear correction terms to NG

The expansion of the CMB perturbed scalar random field $\Phi(\mathbf{n})$ around a Gaussian random field $\varphi_G(\mathbf{n})$

$$\Phi(\mathbf{n})=\varphi_G(\mathbf{n})+fNL^{(local)}(\varphi_G^{\,2}(\mathbf{n})-\langle\varphi_G^{\,2}(\mathbf{n})\rangle)+gNL^{(local)}(\varphi_G^{\,3}(\mathbf{n})-\langle\varphi_G^{\,3}(\mathbf{n})\rangle)+...,$$

Planck 2015, is compatible with a vanishing non-Gaussianity, provided that the bi-spectrum term is

$fNL^{(local)} = 0.8 \pm 5.0$

Conclusions

CMB non-Gaussianity is better fitted by model-independent expansion than PT expansion.

The CMB, dipole-corrected, observed through a homogeneous and isotropic expanding Universe and perturbatively expanded over the 2-sphere of constant curvature, is weakly non-Gaussian and nearly compatible with the prescriptions of the Standard Model with single field Inflation... But there are open questions,

Can we reconstruct model-independently the hidden 27% CMB?

Can we decide what is the kinematic contribution to the dipole?

What if we interpret a part of the dipole and the CMB temperature anisotropy as a distorsion of the spherical support manifold?

Dziekuje